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DESIGN OF A ROBUST FUZZY POWER SYSTEM STABILIZER

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Abstract

To enhance the performance and widen the operating range, the design of a fuzzy observer-based power system stabilizer (PSS) is presented in this paper. A new practical design, which guarantees robust pole-clustering in an acceptable region in the complex plane for a wide range of operating conditions, is proposed. The nonlinear power system model with uncertainties is initially approximated by a Takagi-Sugeno (T-S) fuzzy system. A fuzzy observer/regulator based on a parallel distributed compensation (PDC) control law is suggested. Sufficient design conditions are derived in the form of linear matrix inequalities (LMI). The design procedure leads to a convex optimization problem in terms of the observer and stabilizer gain matrices. Simulations results of both single-machine and multimachine power systems confirm the effectiveness of the proposed PSS design.

Key Words

Power system stability, robustness, LMI, T-S fuzzy model, PDC, fuzzy observer/regulator

1. Introduction

Recently, fuzzy logic has emerged as a potential technique for PSS design [1–3]. Besides its ability to accommodate the heuristic knowledge of a human expert, the advantage of a fuzzy PSS is that it represents a nonlinear mapping that can cope with the nonlinear nature of power systems. Several results confirm that a fuzzy PSS outperforms a conventional PSS once the deviation from the nominal design conditions becomes significant [1]. Up till now, fuzzy logic control applications in power systems are originally introduced and developed as model-free control systems. Although the performance of a model-free fuzzy PSS is acceptable, it unfortunately suffers form lacking of systematic stability analysis and controller synthesis. This paper attempts to overcome this drawback by providing a model-based fuzzy control system that guarantees robust stability and performance of a power system over wide range of operating conditions. In the past 10 years or so, search efforts on fuzzy logic control have been devoted to model-based fuzzy control systems [4]. Stability and performance limits of model-based fuzzy control systems are assessed via linear matrix inequality (LMI) techniques, e.g. [5].

An LMI design of a fuzzy observer-based PSS is proposed in this paper. The proposed design guarantees robust pole-clustering in a prespecified LMI region to guarantee adequate damping characteristics over wide range of operating conditions. Power system design model is described by a polytopic Takagi-Sugeno (T-S) fuzzy system. According to the universal approximation theorem [5], a T-S fuzzy model can approximate the original nonlinear system to an arbitrary degree of accuracy. A T-S fuzzy model allows us to use an imprecise design model [6, 7]. It also enables a decentralized design approach that is independent of the power system size. Up to our knowledge, application of a model-based fuzzy control in power system stabilization, as proposed here, is a novel approach that combines T-S fuzzy model and an observer-based design of a model-based fuzzy PSS.

The rest of paper is organized as follows. Section 2 describes a polytopic model of single-machine infinite-bus (SMIB) system to allow for various operating points. In Section 3, a T-S fuzzy model is proposed for PSS design. In Section 4, sufficient LMI constraints of pole-clustering are recalled. Sufficient LMI constraints for synthesizing fuzzy observer-based PSS, that insure D-stability of a power system, are derived and presented in Section 5. In Section 6, simulation results illustrate the merits of the proposed design. Section 7 concludes this work.

2. Problem Formulation

Power systems consist of highly nonlinear interconnected subsystems. The interconnection occurs via a transmission network by active and reactive power generation of each subsystem (P, Q). Each subsystem comprises one machine connected to the rest of the system by a tie-line whose reactance is the self reactance at the generator bus $X_e = X_{Th}$. For modelling and design approaches proposed here, a subsystem is considerably approximated by a single-machine infinite-bus (SMIB) system. This approximation is made possible because fuzzy modelling allows imprecision [6, 7]. As a result of this assumption, each

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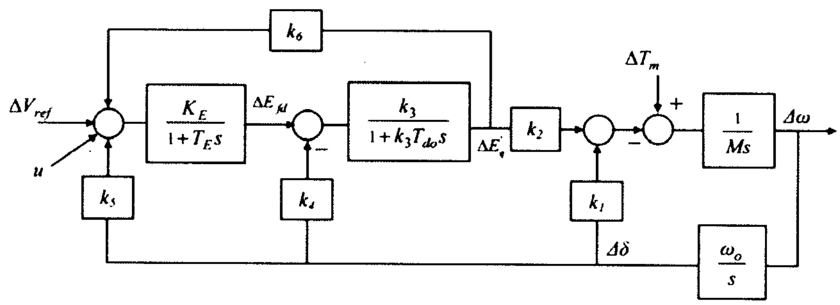

Figure 1. Linearized model of an SMIB system [8].

generator can be decoupled from the rest of the system and consequently leads to a decentralized design and results in reduced order PSSs.

An SMIB system is represented by a 4th order model and the block diagram of the linearized model as described in [8] is shown in Fig. 1. The k-parameters of the block diagram depend explicitly on the generation (P, Q) and tie-line reactance X_e [9]. The variables (P, Q, X_e) are assumed to vary independently over $[\overset{-}{P}\ \overset{+}{P}]$, $[\overset{-}{Q}\ \overset{+}{Q}]$, $[\overset{-}{X_e}\ \overset{+}{X_e}]$ respectively. These ranges are selected to encompass all practical operating points and very weak to very strong transmission networks. State space realization of the linearized system takes the following form:

$$\dot{x} = Ax + Bu, \quad y = Cx \tag{1}$$

The state vector is composed of the deviations in power angle $\Delta\delta$, rotor speed $\Delta\omega$, induced voltage $\Delta E_q^{'}$, and excitation voltage ΔE_{fd}, i.e. $x = [\Delta\delta\ \Delta\omega\ \Delta E_q^{'}\ \Delta E_{fd}]^T$. The purpose of a PSS is to provide a stabilizing signal u such that the speed deviation $\Delta\omega$ would vanish with an acceptable transient behaviour following disturbances. Considering the block diagram in Fig. 1, the matrices A, B and C are given by:

$$A = \begin{bmatrix} 0 & \omega_o & 0 & 0 \\ -\frac{k_1}{M} & 0 & -\frac{k_2}{M} & 0 \\ -\frac{k_4}{T_{do}'} & 0 & \frac{-1}{T_{do}' k_3} & \frac{1}{T_{do}'} \\ -\frac{K_E k_5}{T_E} & 0 & -\frac{K_E k_6}{T_E} & -\frac{1}{T_E'} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 & \frac{K_E}{T_E} \end{bmatrix}^{\mathrm{T}}, C = [0\ 1\ 0\ 0] \tag{2}$$

The symbols in A and B have been defined in [8]. It is noted that A depends on the so-called k-parameters $(k_1, \ldots, k_6)$ of the model shown in Fig. 1. As P, Q and X_e vary, the matrix A varies also. The proposed PSS should stabilize the system over all possible operating points within the ranges $[\overset{-}{P}, \overset{+}{P}]$, $[\overset{-}{Q}, \overset{+}{Q}]$ and $[\overset{-}{X_e}\ \overset{+}{X_e}]$. A T-S fuzzy model is proposed in Section 3 for sake of design. The T-S fuzzy system comprises a set of IF–THEN rules. The consequents of the rules comprise eight linearized models, i.e. (A_i, B, C), $i = 1, \ldots, 8$. These models are respectively calculated at possible combinations of the operating ranges, i.e. $\{\overset{-}{P}, \overset{-}{Q}, \overset{-}{X_e}\}$, $\{\overset{-}{P}, \overset{-}{Q}, \overset{+}{X_e}\}$, $\{\overset{-}{P}, \overset{+}{Q}, \overset{-}{X_e}\}$, $\{\overset{-}{P}, \overset{+}{Q}, \overset{+}{X_e}\}$, $\{\overset{+}{P}, \overset{-}{Q}, \overset{-}{X_e}\}$, $\{\overset{+}{P}, \overset{-}{Q}, \overset{+}{X_e}\}$, $\{\overset{+}{P}, \overset{+}{Q}, \overset{-}{X_e}\}$, $\{\overset{+}{P}, \overset{+}{Q}, \overset{+}{X_e}\}$. So, the resulting rule-base consists of eight rules only. These rules represent the vertices of a polytopic model where each vertex corresponds to one of the models included in the rule-base. The resulting problem corresponds to stabilizing eight plants simultaneously. This can be achieved using the parallel distributed compensation algorithm as explained in Section 3. This is a crucial result as it means that, by stabilizing the fuzzy model, we actually stabilize every model that lies within the polytope. Consequently, as long as $P \in [\overset{-}{P}, \overset{+}{P}]$, $Q \in [\overset{-}{Q}, \overset{+}{Q}]$ and $X_e \in [\overset{-}{X_e}\ \overset{+}{X_e}]$, the proposed design must guarantee D-stability of a power system.

3. An Equivalent T-S Fuzzy Model for a Power System

3.1 Review of T-S Fuzzy Model and PDC

T-S fuzzy model is in fact a fuzzy dynamic model [5–7]. This model is based on using a set of fuzzy rules to describe a global nonlinear system by a set of local linear models which are smoothly connected by fuzzy membership functions. There are two basic approaches to identify T-S fuzzy models [5], one is to linearize the original nonlinear system in a number of operating points when the model is known, which is adopted in this study. The second is based on the data gathered from the nonlinear system when the model is unknown. The ith rule of a T-S fuzzy model is written as follows:

$$\begin{gathered} \text{IF } z_1(t) \text{ is } M_1^i\ AND \ldots AND\ z_n(t) \text{ is } M_n^i \\ \text{THEN } \dot{x}(t) = A_i x(t) + B_i u(t), \quad y(t) = C_i x(t) \end{gathered}$$

where M_j^i, $j = 1, \ldots, n$, is the jth fuzzy set of the ith rule and $z_1(t), \ldots, z_n(t)$ are known as premise variables. Let $\mu_j^i(z_j)$ be the membership function of the fuzzy set M_j^i and let $h_i = h_i(t) = \Pi_{j=1}^n \mu_j^i(z_j)$. Therefore, resulting T-S fuzzy system is inferred as the weighted average of the local models as follows:

$$\begin{aligned} \dot{x}(t) &= \frac{\sum_{i=1}^r h_i\{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^r h_i} \\ h_i &= \sum_{i=1}^r \alpha_i\{A_i x(t) + B_i u(t)\} \\ y(t) &= \sum_{i=1}^r \alpha_i\{C_i x(t)\} \end{aligned} \tag{3}$$

where $\alpha_i = h_i/\sum_{i=1}^{r} h_i$ are the normalized weights and $\sum_{i=1}^{r} \alpha_i = 1, \alpha_i \geq 0, i = 1, \ldots, r$.

PDC offers a procedure to design a fuzzy controller from a given T-S fuzzy model [5, 10]. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts. Therefore, a fuzzy state feedback regulator for the T-S fuzzy model (3) can be constructed via PDC as follows:

Model Rule # i:
IF $z_1(t)$ *is* M_1^i *AND* $\ldots$ *AND* $z_n(t)$ *is* M_n^i
THEN $u(t) = F_i x(t), \quad i = 1, \ldots, r$

The overall fuzzy controller is given by:

$$u(t) = \frac{\sum_{i=1}^{r} h_i\{F_i x(t)\}}{\sum_{i=1}^{r} h_i} = \sum_{i=1}^{r} \alpha_i\{F_i x(t)\} \tag{4}$$

Although the fuzzy controller (4) is constructed using local design structures, the feedback gains must be determined using global design conditions to guarantee global stability and performance. Different methods for stability analysis and control design of T-S fuzzy systems are reported in [4]. Closed loop stability of a T-S fuzzy system is enforced here by insuring stability of all local models using common Lyapunov matrix. Combining (3) and (4), the closed loop system is given by:

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \alpha_i \alpha_j \{(A_i + B_i F_j)x(t)\} \tag{5}$$

If $B_i = B$, $i = 1, \ldots, r$, (5) can be rewritten as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} \alpha_i \{(A_i + BF_i)x(t)\} \tag{6}$$

Theorem 1 [5]: The T-S fuzzy model (6) is globally asymptotically stable if there exists a common positive definite matrix X such that:

$$(A_i + BF_i)^T X + X(A_i + BF_i) < 0, \quad i = 1, \ldots, r \tag{7}$$

Remark 1: Theorem 1 presents sufficient stability conditions of a T-S fuzzy system having a common B matrix. The eight local models of the T-S fuzzy system proposed in Section 3.2 have the same B matrix since the parameters K_E and T_E of the excitation system are fixed.

3.2 Deriving the T-S Fuzzy Model for the Proposed PSS Design

3.2.1 Rule-Base Creation

Each vertex system in the polytope presented at the end of Section 2 corresponds to an IF-THEN rule in a T-S fuzzy system which is stated as follows:

Model Rule 1: IF $[P \text{ is about } \overset{-}{P}]$ AND $[Q \text{ is about } \overset{-}{Q}]$ AND $[X_e \text{ is about } \overset{-}{X_e}]$ THEN $\dot{x} = A_1 x + Bu, y = Cx$

Model Rule 2: IF $[P \text{ is about } \overset{-}{P}]$ AND $[Q \text{ is about } \overset{-}{Q}]$ AND $[X_e \text{ is about } \overset{+}{X_e}]$ THEN $\dot{x} = A_2 x + Bu, y = Cx$

$, \ldots,$

Model Rule 8: IF $[P \text{ is about } \overset{+}{P}]$ AND $[Q \text{ is about } \overset{+}{Q}]$ AND $[X_e \text{ is about } \overset{+}{X_e}]$ THEN $\dot{x} = A_8 x + Bu, y = Cx$

Only the boundaries of (P, Q, X_e) are considered to develop the T-S fuzzy model where the consequents of rule-base (local models) are calculated at these boundaries. Physically, these boundaries comprise all practical extreme operating conditions involved during a power system operation. The T-S fuzzy model involves a polytope where the linearized models of the rule consequents represent its vertices. Sufficiently, fuzzy aggregation of these eight local models can account for any operating point as long as it lies within this polytope and the design approach is valid for such a point.

3.2.2 Membership Functions

Any value $P \in [\overset{-}{P} \;\; \overset{+}{P}]$ can be expressed as $P = L_1(\overset{-}{P}, \overset{+}{P}, P) \times \overset{-}{P} + L_2(\overset{-}{P}, \overset{+}{P}, P) \times \overset{+}{P}$, where $L_1(\overset{-}{P}, \overset{+}{P}, P)$ and $L_2(\overset{-}{P}, \overset{+}{P}, P)$ are membership functions for the variable P such that $L_1(\overset{-}{P}, \overset{+}{P}, P) + L_2(\overset{-}{P}, \overset{+}{P}, P) = 1$. Consequently, these membership functions can be calculated as follows:

$$L_1(\overset{-}{P}, \overset{+}{P}, P) = \frac{(\overset{+}{P} - P)}{(\overset{+}{P} - \overset{-}{P})} \quad L_2(\overset{-}{P}, \overset{+}{P}, P) = \frac{(P - \overset{-}{P})}{(\overset{+}{P} - \overset{-}{P})}$$

The membership functions $L_1(\overset{-}{P}, \overset{+}{P}, P)$ and $L_2(\overset{-}{P}, \overset{+}{P}, P)$ are labelled "$\overset{-}{P}$" and "$\overset{+}{P}$" respectively. Similarly, membership functions for Q and X_e are labelled (M_1, M_2), (N_1, N_2) respectively and given by:

$$M_1(\overset{-}{Q}, \overset{+}{Q}, Q) = \frac{\overset{+}{Q} - Q}{\overset{+}{Q} - \overset{-}{Q}}, \quad M_2(\overset{-}{Q}, \overset{+}{Q}, Q) = \frac{Q - \overset{-}{Q}}{\overset{+}{Q} - \overset{-}{Q}},$$

$$N_1(\overset{-}{X_e}, \overset{+}{X_e}, X_e) = \frac{\overset{+}{X_e} - X_e}{\overset{+}{X_e} - \overset{-}{X_e}}, \quad N_2(\overset{-}{X_e}, \overset{+}{X_e}, X_e) = \frac{X_e - \overset{-}{X_e}}{\overset{+}{X_e} - \overset{-}{X_e}}$$

The weights are given by: $h_1 = L_1 M_1 N_1$, $h_2 = L_1 M_1 N_2$, $h_3 = L_1 M_2 N_1, \ldots, h_8 = L_2 M_2 N_2$ and the normalized weights or the firing strengths are given by $\alpha_i = h_i/\sum_{i=1}^{8} h_i$, $i = 1, \ldots, 8$. It is easily proved that $\sum_{i=1}^{8} \alpha_i = 1$. The resulting system could be inferred as the weighted average of the local models as follows:

$$\dot{x} = \sum_{i=1}^{8} \alpha_i \{A_i x\} + Bu, y = Cx \tag{8}$$

Equation (8) describes the result of defuzzification and aggregation of the T-S fuzzy system presented in Section 3.2.1, i.e. (8) gives the inferred system in terms of eight local models and a set of firing strengths. Briefly, the state-space model of a certain operating point (P, Q, X_e) is expressed as a convex combination of these local models as long as $P \in [\overset{-}{P}, \overset{+}{P}]$, $Q \in [\overset{-}{Q}, \overset{+}{Q}]$ and $X_e \in [\overset{-}{X_e}\ \overset{+}{X_e}]$. Further, the controller for such a point is expressed as convex combination of the eight local controllers designed for local models considering the same firing strengths.

4. Representing Power System Specifications as an LMI Region

Sufficient damping can be insured by clustering the closed loop poles in an admissible region as shown in Fig. 2. This ensures a minimum damping factor of α_R and a minimum damping ratio $\zeta_{\min} = \cos(\Theta/2)$. This in turn bounds the maximum overshoot and the settling time of the closed loop system. To avoid very large observer and feedback gains, the real part of the poles should be placed to the right of the α_L-line.

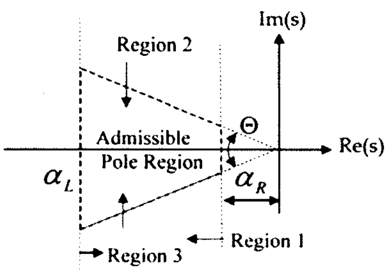

Figure 2. Considered LMI region.

This admissible pole region can be expressed as an LMI region defined by three individual LMI regions as shown in Fig. 2. An LMI region is any subset D of the complex plane C that has been defined in [11] as follows:

$$D = \{s \in C\colon \Phi + s\Psi + \bar{s}\Psi^T < 0\} \tag{9}$$

where Φ and Ψ are real matrices and $\Phi = \Phi^T$. The region matrices Φ and Ψ are calculated from the values of α_R, α_L and Θ as reported in [11]. The following lemma gives sufficient LMI condition to guarantee D-stability of an autonomous system.

Lemma 1 [11]. The system $\dot{x}(t) = Ax(t)$ is D-stable if and only if a symmetric, positive definite matrix X exists such that: $\Phi \otimes X + \Psi \otimes (XA) + \Psi^T \otimes (XA)^T < 0$, where $\otimes$ is the Kronecker product.

5. Design of the Proposed Fuzzy Observer-Based Stabilizer

Typically, a PSS has the speed deviation as a feedback signal. In such case, attention is oriented towards output feedback design methods such as an observer-based design. This section presents an algorithm for state estimation of T-S fuzzy models to implement a pole-clustering observer-based stabilizer. A fuzzy observer is constructed such that it shares the same fuzzy sets with the fuzzy model [5, 12] as follows:

Observer Rule i:

IF $z_1(t)$ is M_1^i $AND \ldots AND$ $z_n(t)$ is M_n^i THEN
$\dot{\tilde{x}}(t) = A_i\tilde{x}(t) + B_i u(t) + K_i(y(t) - \tilde{y}(t)), \tilde{y}(t) = C_i\tilde{x}(t)$

The fuzzy observer is represented as follows:

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r} \alpha_i \{A_i\tilde{x}(t) + B_i u(t) + K_i(y(t) - \tilde{y}(t))\},$$
$$\tilde{y}(t) = \sum_{i=1}^{r} \alpha_i \{C_i\tilde{x}(t)\} \tag{10}$$

If the fuzzy observer exists, the fuzzy state feedback regulator takes the following form:

$$u(t) = \sum_{i=1}^{r} \alpha_i F_i \tilde{x}(t) \tag{11}$$

Combining (10) and (11), then denoting $e(t) = x(t) - \tilde{x}(t)$, the error dynamics are represented as follows:

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \alpha_i\alpha_j\{(A_i + B_iF_j)x(t) - B_iF_je(t)\} \tag{12}$$
$$\dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \alpha_i\alpha_j\{(A_i - K_iC_j)e(t)\} \tag{13}$$

. The closed loop system can be written as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \sum_{i=1}^{r}\sum_{j=1}^{r} \alpha_i\alpha_j \left\{ \begin{bmatrix} A_i + B_iF_j & -B_iF_j \\ 0 & A_i - K_iC_j \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \right\} \tag{14}$$

For the case under study, $B_i = B$, $C_j = C$, $i, j = 1, \ldots, r$, then (14) can be rewritten as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \sum_{i=1}^{r} \alpha_i \left\{ \begin{bmatrix} A_i + BF_i & -BF_i \\ 0 & A_i - K_iC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \right\} \tag{15}$$

The stability of the closed loop system (15) is analyzed through the following lemma and theorem.

Lemma 2. Let $\overline{A} = \sum_{i=1}^{r} \alpha_i\tilde{A}_i$, where $\overline{A}, \tilde{A}_i \in \Re^{n\times n}$ and $0 \le \alpha_i \le 1, \sum_{i=1}^{r} \alpha_i = 1$. The eigenvalues of $\overline{A}$ lies in the LMI region $D = \{s \in C\colon \Phi + s\Psi + \bar{s}\Psi^T < 0\}$ if a positive definite matrix X exists such that the following

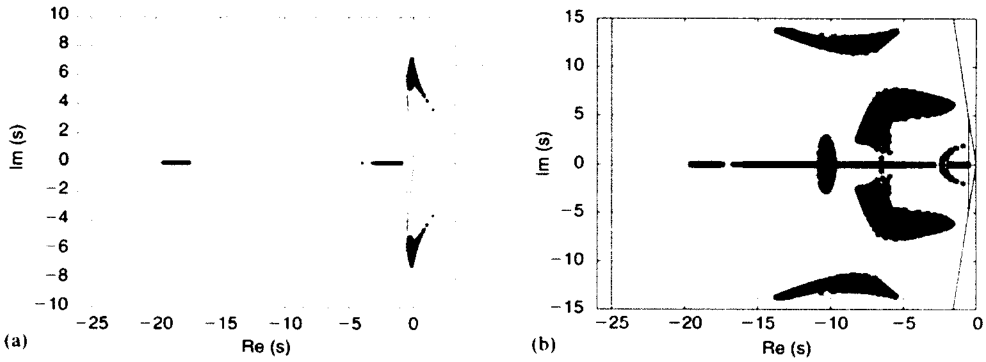

Figure 3. System roots: (a) Uncontrolled system and (b) Controlled system.

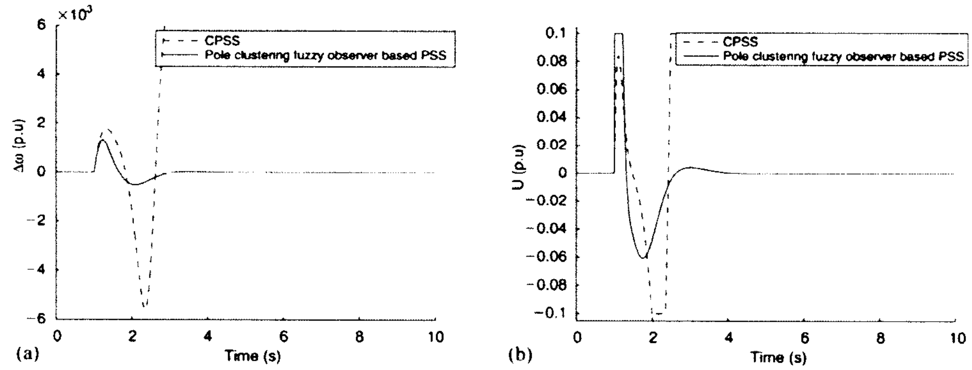

Figure 4. 10% step increment in T_m at $P=1.0$, $Q=-0.2$, $X_e=0.4$: (a) Speed deviation (p.u) and (b) Control signal (p.u).

LMIs are satisfied $\Phi \otimes X + \Psi \otimes (\tilde{A}_i X) + \Psi^T \otimes (\tilde{A}_i X)^T < 0$, $i=1,\ldots,r$.

Proof: The proof follows immediately from the convexity of the region D, the summation form of $\overline{A}$, and Lemma 1. Let $f_D(\tilde{A}_i, X) = \Phi \otimes X + \Psi \otimes (\tilde{A}_i X) + \Psi^T \otimes (\tilde{A}_i X)^T < 0$, $i=1,\ldots,r$, it follows that $\sum_{i=1}^{r} \alpha_i f_D(\tilde{A}_i, P) < 0$. Since $\sum_{i=1}^{r} \alpha_i f_D(\tilde{A}_i, X) = f_D(\sum_{i=1}^{r} \alpha_i \tilde{A}_i, X)$, it is possible to conclude that $f_D(\overline{A}, X) < 0$. This means that the eigenvalues of $\overline{A}$ lie inside the region D. □

Hint. Lemma 2 states that if a set of matrices have their eigenvalues in an LMI region; their convex sum will also have its eigenvalues inside the same LMI region. The above LMI could be also stated as $\Phi \otimes X + \Psi \otimes (X\tilde{A}_i) + \Psi^T \otimes (X\tilde{A}_i)^T < 0$, $i=1,\ldots,r$, [11]. This form is used in the observer design while the form that appears in Lemma 2 is used for regulator design.

Theorem 2. Let $F_i = M_i X_1^{-1}$ and $K_i = X_2^{-1} N_i$, $i = 1,\ldots,8$. The eigenvalues of the closed loop system (15) lie in the LMI region given by $D = \{s \in C\colon \Phi + s\Psi + \overline{s}\Psi^T < 0\}$ if there exist real matrices M_i, N_i, $X_1 > 0$, and $X_2 > 0$ such that the following LMIs are satisfied:

$$
\begin{aligned}
&\Phi \otimes X_1 + \Psi \otimes (A_i X_1 + BM_i) + \Psi^T \\
&\quad \otimes (A_i X_1 + BM_i)^T < 0 \; i = 1,\ldots,8 \\
&\Phi \otimes X_2 + \Psi \otimes (X_2 A_i - N_i C) + \Psi^T \\
&\quad \otimes (X_2 A_i - N_i C)^T < 0 \; i = 1,\ldots,8
\end{aligned} \tag{16}
$$

Proof: The eigenvalues of the closed loop system (15) lie in the LMI region D if the eigenvalues of the matrices $\sum_{i=1}^{r} \alpha_i (A_i + BF_i)$ and $\sum_{i=1}^{r} \alpha_i (A_i - K_i C)$ lie in D. Applying Lemma 2 to $\sum_{i=1}^{r} \alpha_i (A_i + BF_i)$ and $\sum_{i=1}^{r} \alpha_i (A_i - K_i C)$ and using the substitutions $F_i = M_i X_1^{-1}$ and $K_i = X_2^{-1} N_i$, respectively, lead to the LMIs in (16). □

The design steps can be summarized as follows: (1) Determine the ranges $P \in [\overset{-}{P}\ \overset{+}{P}]$, $Q \in [\overset{-}{Q}\ \overset{+}{Q}]$ and $X_e \in [\overset{-}{X_e}\ \overset{+}{X_e}]$ that encompass all practical operating conditions. (2) Define the eight subsystems by calculating $A_1, A_2, \ldots, A_8$, B and C. (3) Define the membership functions according to their shapes and the ranges of P, Q and X_e. (4) Generate the T-S fuzzy system defined by (8). (5) Select α_R, α_L and Θ, then compute the LMI region matrices Φ and ψ as reported in [11]. (6) Find the gains of a robust pole-placement fuzzy observer-based PSS by solving the optimization problem in (16) using an appropriate LMI solver, e.g. [13].

6. Simulation Results

6.1 Single-Machine Infinite-Bus Test System

The study in this section will be carried on a single-machine infinite-bus system whose model and data are given in [9]. The powers at generator terminals and tie-line reactance P, Q and X_e vary independently over: $P \in [0.4\ 1.0]$, $Q \in [-0.2\ 0.5]$ and $X_e \in [0.2\ 0.4]$ (p.u). Fig. 3(a) shows the system open loop poles for 1000 plants as P, Q and X_e vary over their specified ranges. It is noted that most of these plants do not have adequate damping and some plants are unstable. The design of a fuzzy observer-based stabilizer is carried out for an LMI region bounded by $\alpha_L = -25$, $\alpha_R = -0.5$ and $\Theta = 168°$. The matrices ϕ, ψ are given in Appendix A. The LMIs (16) are solved to calculate the observer and regulator gains K_i and F_i, $i = 1, \ldots, 8$. The gains are given in Appendix B. Fig. 3(b) shows the efficacy of the fuzzy observer-based PSS in clustering the system roots in the prespecified LMI region.

The proposed PSS is compared to the conventional PSS (CPSS) given in [14] for the same machine parameters. The system response due to 10% step increment in the mechanical torque (T_m) is depicted in Fig. 4(a). As seen, the proposed fuzzy PSS achieves good transient performance as enforced by the LMI region constraint. The control signal is depicted in Fig. 4(b).

6.2 Decentralized Application in Multimachine Power System

The benchmark model shown in Fig. 5 is adopted for multimachine simulation studies. It is specifically designed in [15] to study low frequency electromechanical oscillations in large interconnected power systems. A general procedure to design a PSS for each generator separately includes the following steps:

i. The load flow study is carried out for different loading conditions that may be encountered during the power system operation to obtain the ranges $P_i \in [\overset{-}{P_i}\ \overset{+}{P_i}]$ and $Q_i \in [\overset{-}{Q_i}\ \overset{+}{Q_i}]$ for different generators, where $i = 1, \ldots, m$ and m is the number of involved generators.
ii. For different network topologies, the bus impedance matrix is calculated, and different self impedances are determined at the generator buses to get $X_i \in [\overset{-}{X_i}\ \overset{+}{X_i}]$, where $i = 1, \ldots, m$.
iii. Once all ranges are determined, the steps described at the end of Section 5 are used to find a T-S fuzzy observer/regulator for each generator separately.

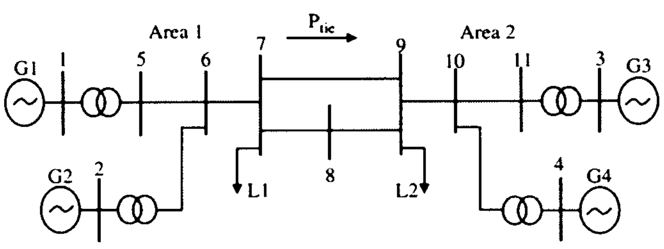

Figure 5. Two-area four-machine test system [15].

Rotor speed of Machine 1 is depicted in Fig. 6 for a three-phase short circuit at middle of one tie-line (Bus 8) when a 430 MW is transferred from Area 1 to Area 2; the fault is cleared after 0.133 s by opening the two breakers at the ends of the faulted line causing one tie-line separation. It is clear that the proposed PSS outperforms CPSS even at the nominal point. The system response under heavy tie-line power is demonstrated by a three phase short circuit on one tie-line at 7 km far from Bus 7, when a tie power of 710 MW is transferred, with full recovery after 0.133 s. The results depicted in Fig. 7(a) show better damping characteristics by the proposed stabilizer while CPSS fails to maintain stability. Fig. 7(b) depicts the tie-line power while Fig. 7(c) shows the control signals of the four machines as supplemented by the proposed stabilizer.

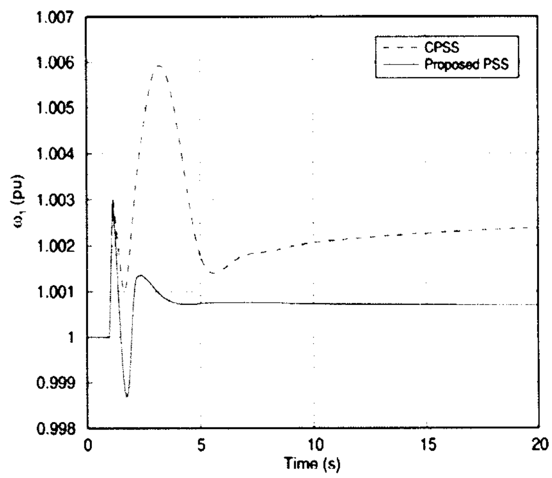

Figure 6. Rotor speed of Machine 1 due to three-phase short circuit at Bus 8 cleared after 0.133 s.

7. Conclusion

This paper has provided a step towards the design of a power system stabilizer that can cope with a wide range of loading conditions and external disturbances. It has been shown that the nonlinear model of a power system can be systematically represented in the form of a T-S fuzzy system. This has allowed us to use an approximate design model of the power system to carry out the design. A fuzzy observer has been designed to estimate the system states assuming speed measurements only. LMI conditions have been derived to facilitate the design of the observer and regulator gains such that the closed loop poles lie in a prespecified LMI region. The resulting design enjoys the separation principle property since the LMI conditions for the observer and regulator are independent. The combined design of fuzzy observer and fuzzy regulator in the PSS framework has been another contribution in the present work. Simulation results of a two-area four-machine power system have confirmed the superiority of the proposed algorithm in damping the post-fault inter-area oscillations. Compared to a well-tuned CPSS, it has been shown that the proposed PSS is less sensitive to the fault location and has a superior capability to cope with heavy tie-line power transfer.

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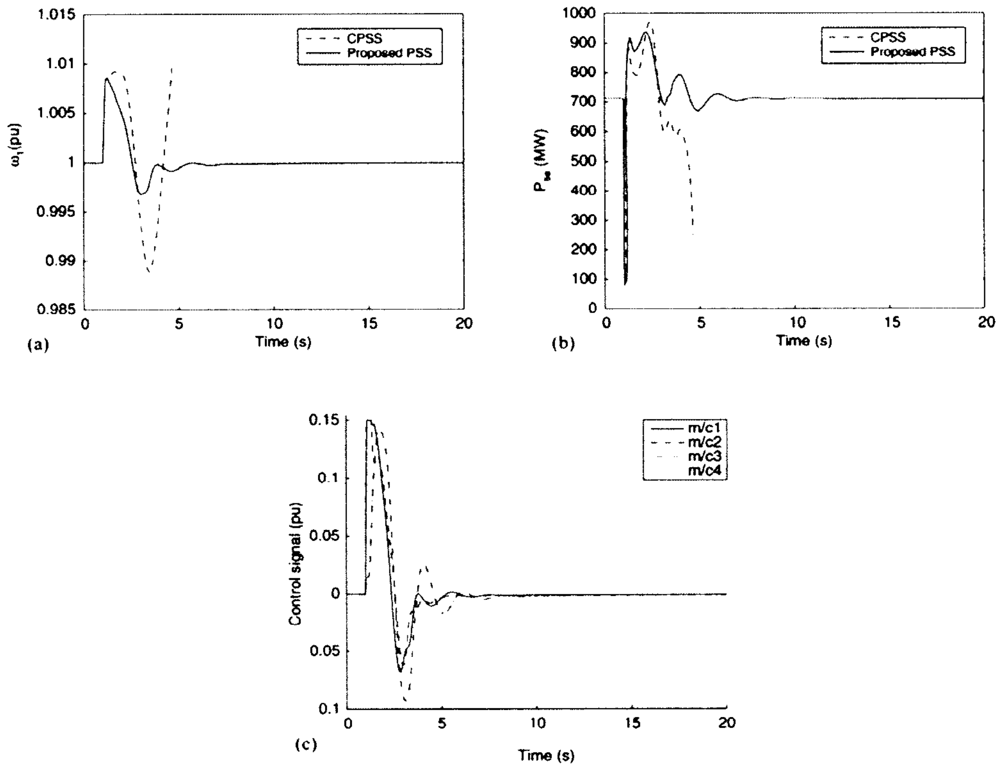

Figure 7. System response due to three-phase short circuit on one tie-line at 7 km far from Bus 7 for 0.133 s: (a) Rotor speed (p.u) of Machine 1, (b) Tie-line power (MW) from Area 1 to Area 2, and (c) Control signals of Machines 1∼4 with the proposed PSSs in (p.u).

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Appendix

A. Matrices of the LMI region, that is bounded by $\alpha_L = -0.5$, $\alpha_R = -25$, $\Theta = 168°$, are given by:

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -50 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Psi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0.99452 & -0.10453 \\ 0 & 0 & 0.10453 & 0.99452 \end{bmatrix}$$

B. Observer and regulator gains are calculated from the optimization problem (16) and given by:

$$K = \left[\begin{matrix} -177.73 & -136.56 & -101.49 & -74.997 \\ 21.675 & 19.823 & 17.554 & 16.845 \\ -132.91 & -116.86 & -99.099 & -86.863 \\ 929.64 & 926.69 & 737.76 & 1092.2 \end{matrix}\right.$$

$$\left.\begin{matrix} -218.39 & -85.966 & -176.79 & -130.72 \\ 21.721 & 18.091 & 20.002 & 18.281 \\ -142.86 & -100.45 & -126.33 & -110.62 \\ -1138.8 & -3266.3 & 144.11 & 107.05 \end{matrix}\right],$$

$$F = \begin{bmatrix} 0.4488 & -230.96 & 4.9786 & 0.016314 \\ 0.3222 & -256.38 & 5.1913 & 0.019055 \\ 0.4319 & -262.86 & 5.0531 & 0.019546 \\ 0.3308 & -272.34 & 4.9689 & 0.020427 \\ 0.8524 & -184.52 & 4.6144 & 0.012201 \\ 0.4481 & -280.13 & 5.8286 & 0.021583 \\ 0.7835 & -201.40 & 4.5245 & 0.013418 \\ 0.5905 & -238.80 & 4.8093 & 0.017006 \end{bmatrix}$$

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